

Date: 12/01/2016

**Época Normal – Block 1**

Duration: **1 hour**

**(Note:** Justify all your answers and present all the calculations)

**Note:** Continuing question 1 from **Block 2 – The LP problem coincides with the one in block 2, the difference is only on the questions you should answer.**

A company aims to maximize the weekly profit with the production and sales of products **A**, **B** and **C**.

To produce **A**, **B** and **C**, 240 hours machine (h.m) are available per week. Prior to selling, products need to be in an oven that has 700 m<sup>3</sup> capacity. The company cannot sell more units of **B** and **C** together than **A**. An agreement forces the production of a minimum of 20 units of **C** every week.

The corresponding LP formulation follows:

$$\begin{aligned} \max Z &= 10x_A + 20x_B + 40x_C \\ \text{s. t. : } &\begin{cases} x_A + x_B + 4x_C \leq 240 \\ 4x_A + 2x_B + x_C \leq 700 \\ x_A - x_B - x_C \geq 0 \\ x_C \geq 20 \\ x_A, x_B, x_C \geq 0 \end{cases} \end{aligned}$$

By Solver/Excel the following “Sensitivity Report” was obtained:

**Microsoft Excel 15.13 Sensitivity Report**

**Variable Cells**

| Cell   | Name  | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|--------|-------|-------------|--------------|-----------------------|--------------------|--------------------|
| \$B\$7 | $x_A$ | 90          | 0            | 10                    | 10                 | 16,6667            |
| \$C\$7 | $x_B$ | 70          | 0            | 20                    | 1E+30              | 10                 |
| \$D\$7 | $x_C$ | 20          | 0            | 40                    | 25                 | 1E+30              |

**Constraints**

| Cell   | Name                   | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|--------|------------------------|-------------|--------------|----------------------|--------------------|--------------------|
| \$E\$2 | machine (h.m.)         | 240         | 15           | 240                  | 60                 | 140                |
| \$E\$3 | oven (m <sup>3</sup> ) | 520         | 0            | 700                  | 1E+30              | 180                |
| \$E\$4 | sales - A vs B & C     | 0           | -5           | 0                    | 140                | 180                |
| \$E\$5 | sales of C             | 20          | -25          | 20                   | 28                 | 18                 |

- a) (2,0 points) Formulate the dual of the given problem.
- b) (1,0 point) Write and interpret the primal optimal solution only for decision variables.
- c) (1,5 points) Write and interpret the dual optimal solution (decision variables) as well as the slack variables of the primal and relate the corresponding values.
- d) (5,5 points) Considering the formulation given above **without** the fourth constraint (about the sell to the important client):
  - d.1) Write the problem in the augmented form.
  - d.2) Perform one iteration by simplex method. Write and classify the solution obtained.
  - d.3) Identify the basic variables and the non basic variables in the solution found in d.2).
  - d.4) Justify if the optimal value of this problem (without the fourth constraint) can be smaller than the optimal value of the initial problem.